ECS120 Introduction to the Theory of Computation
Fall Quarter 2007

Homework 4 Help
Due Friday October 26, 2007

Problem 4.1

Prove that the following languages are not regular. You can use the fact that $L = \{ 0^n 1^n \mid n \geq 0 \}$ is non-regular.

(a) $\{ 0^n 0^m \mid n \leq m \}$
(b) $\{ 1^{2^n} \mid n = 0, 1, 2, \ldots \}$
(c) $\{ w \mid w \text{ is not a palindrome} \}$
(d) $\{ 0^n 1^m 2^{n-m} \mid n \geq m \geq 0 \}$

See the discussion notes for how to prove a language is not regular. (There are two approaches you can take.)

Problem 4.2

Consider languages over a fixed alphabet Σ with $|\Sigma| = 2$. Prove or disprove the following.

Remember to disprove a statement, you just need to provide a counterexample. Choose a L_1 and L_2 which disproves the statement. Use simple languages that we have already proven to be regular or non-regular. See the discussion notes for an example.

Otherwise, use the general proof mechanisms that we have already used in this class to prove that the statement is true.

Problem 4.3

Consider $F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$. (a) Show that F is not regular, (b) that F acts like a regular language in the pumping lemma, and (c) explain why this does not contradict the pumping lemma.

(Sipser Problem 1.54)

We already talked about different methods for showing that F is not regular. Use method 2 (closure properties) for part (a).

For part (b) do exactly what you are asked. Choose a pumping length of p. Show that for all $w \in F$ such that $|w| \geq p$ the pumping lemma conditions hold. For example, suppose you choose $p = 2$. For all strings w such that $|w| \geq 2$, you must show that each of the pumping lemma conditions hold. (This part may be tricky!)

For part (c) explain why this does not contradict the pumping lemma. Ask yourself why choosing w is so important when proving a language is non-regular with the pumping lemma. What specifically does the pumping lemma tell us? What does it not tell us?

Don't forget to address all three parts of this problem!

Problem 4.4

Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$

(Sipser Exercise 2.4)

Do examples and check your work. If you haven't already noticed, 2.4 (a) and (d) are solved for you in the book. (That is what the $^{\mathsf{A}}$**a.** notation means!) You can find the solutions on page 132 (just before chapter 3).

See the discussion notes for example grammars.