Section 1.1

1. Write the following compound propositions in good English, using the following:

   v: "I take a vacation"
   s: "it is summer"
   w: "I work"

(a) $s \rightarrow v$

(b) $s \rightarrow \neg w$

(c) $w \oplus v$

Solution

(a) I take a vacation in the summer.

(b) I don't work in the summer.

(c) I work or take a vacation, but not both.

2. Write the following compound propositions in symbols, using the following:

   v: "I take a vacation"
   s: "it is summer"
   w: "I work"

(a) I take a vacation only if it is summer.

(b) When it is summer I take a vacation, but when it is not summer I work.

(c) I work and take a vacation every summer.

Solution

(a) $v \rightarrow s$, or, equivalently, $\neg s \rightarrow \neg v$

(b) $(s \rightarrow v) \wedge (\neg s \rightarrow w)$

(c) $s \rightarrow (w \wedge v)$

3. Are the following specifications consistent?

   If sales don't increase, then profits on exports will decrease.

   If unemployment does not drop, then imports rise.

   The economy improves exactly when unemployment drops.

   If profits on exports decrease, then imports do not go up.

   The economy gets worse and sales decrease.

Solution Using the following symbols:

i: "sales increase"
p: "profits on exports decrease"
u: "unemployment drops"
e: "the economy improves"
m: "imports go up"

the given statements can be written in symbols as:

1. $\neg i \rightarrow p$

2. $\neg u \rightarrow m$

3. $e \leftrightarrow u$

4. $p \rightarrow \neg m$

5. $\neg e \wedge \neg i$ (In this statement, saying that the economy gets worse and sales decrease allows us to say that the economy does not improve and sales do not increase, that is, $\neg e \wedge \neg i$.)

By statement 5, $\neg e$ is true, and hence e is false. This forces $\neg u$ to be true by statement 3. This forces m to be true by statement 2, and hence $\neg m$ must be false. Therefore, by statement 4, p must be false. By statement 1, $\neg i$ must also be false, and hence i is true. But $\neg i$ is true because of statement 5. Thus, the specifications are not consistent.

4. Here is a newspaper headline:

"Legislature Fails to Override Governor's Veto of Bill to Cancel Sales Tax Reform".

Did the legislature vote in favor of or against sales tax reform?

Solution Let s stand for "sales tax reform is supported". Then the bill to cancel sales tax reform is $\neg s$, and the governor's veto of this bill is $\neg\neg s$. Overriding this would be $\neg\neg\neg s$, and failing to override is $\neg\neg\neg\neg s = s$. Therefore, the headline is equivalent to s, and hence the Legislature supports sales tax reform.