Section 1.3

1. Write the following statements in good English. Use the following variables and predicates:

$$\begin{aligned} x&: \text{ people} \\ y&: \text{ stores} \\ S(x,y)&: \text{ ``}x\text{ shops in }y\text{''} \\ T(x)&: \text{ ``}x\text{ is a student''} \end{aligned}$$

   (a) $\forall y\, S(Margaret, y)$
   (b) $\exists y\, \forall x\, S(x, y)$
   (c) $\forall x\, \exists y\, S(x, y)$
   (d) $\exists y\, \forall x\, [T(x) \to \neg S(x, y)]$
   (e) $\forall y\, \exists x\, [T(x) \wedge S(x, y)]$

   Solution

   (a) Margaret shops in every store.
   (b) There is a store in which everyone shops.
   (c) Everyone shops somewhere.
   (d) There is a store in which no student shops.
   (e) Every store has at least one student who shops in it.

2. Write the following statements in good English. Use the following variables and predicates:

$$\begin{aligned} x&: \text{ people} \\ y&: \text{ stores} \\ S(x,y)&: \text{ ``}x\text{ shops in }y\text{''} \\ T(x)& \quad \text{ ``}x\text{ is a student''} \end{aligned}$$

   (a) Will shops in Al's Record Shoppe.
   (b) There is no store that has no students who shop there.
   (c) The only shoppers in some stores are students.

   Solution

   (a) $S(\text{Will, Al's Record Shoppe})$
   (b) $\neg\exists y\, \forall x\, [T(x) \to \neg S(x, y)]$
   (c) $\exists y\, \forall x\, [S(x, y) \to T(x)]$

3. Write the following statements in good English. Use the following variables and predicates:

x:	students
y:	courses
$F(x)$:	"x is a Freshman"
$C(x)$:	"x is a Computer Science major"
$M(y)$:	"y is a math course"
$T(x, y)$:	"x is taking y

(a) $C(\text{Ben})$

(b) $\exists x\,[F(x) \wedge T(x, \text{Calculus III})]$

(c) $\forall x\,\exists y\,[C(x) \rightarrow M(y) \wedge T(x, y)]$

(d) $\forall y\,\exists x\,[\neg(M(y) \wedge T(x, y))]$

(e) $\neg\exists x\,[F(x) \wedge \forall y\,[M(y) \rightarrow T(x, y)]]$

Solution

(a) Ben is a Computer Science major.

(b) Some Freshman is taking Calculus 3.

(c) Every Computer Science major is taking at least one math course.

(d) Every course has a student in it who is not a Math major

(e) No Freshman is taking every math course.

4. Consider the following lines of code from a C++ program:

```
if (!(x!=0 && y/x < 1) || x==0)
cout << "True";
else
cout << "False"
```

(a) Express the code in this statement as a compound statement using the logical connectives $\neg, \vee, \wedge, \rightarrow$, and the following predicates

$E(x)$:	$x = 0$
$L(x, y)$:	$y/x < 1$
$A(z)$:	"z is assigned to `cout`"

where x and y are integers and z is a Boolean variable (with values True and False).

(b) Use the laws of propositional logic to simplify the statement by expressing it in a simpler form.

(c) Translate the answer in part (b) back into C++.

Solution

(a) First we insert the predicates into the code, obtaining

```
if (!(!E(x) && L(x,y)) || E(x))
A(True)
else
A(False).
```

Next change to the usual logical connective symbols, keeping in mind that C++ code of the form "if p then q else r" is really a statement of the form $(p \to q) \wedge (\neg p \to r)$:

$[\neg\,(\neg\, E(x)\;\wedge\; L(x,y)) \vee E(x)] \to$
$A(\text{True})$
$\wedge$
$\neg\,[\neg\,(\neg\, E(x)\;\wedge\; L(x,y)) \vee E(x)] \to$
$A(\text{False})$, or

$$\Big([\neg\,(\neg\, E(x)\,\wedge\, L(x,y)) \vee E(x)] \to A(\text{True})\Big)\;\wedge\;\Big(\neg\,[\neg\,(\neg\, E(x)\,\wedge\, L(x,y)) \vee E(x)] \to A(\text{False})\Big).$$

(b) Using DeMorgan's law on the negation of the conjunction, the statement becomes

$$\Big([(E(x)\;\vee\;\neg L(x,y)) \vee E(x)] \to A(\text{True})\Big)\;\wedge\;\Big(\neg\,[(E(x)\;\vee\;\neg\, L(x,y)) \vee E(x)] \to A(\text{False})\Big),$$

which can be simplified to give

$$\Big((E(x)\;\vee\;\neg L(x,y)) \to A(\text{True})\Big)\;\wedge\;\Big(\neg\,(E(x)\;\vee\;\neg\, L(x,y)) \to A(\text{False})\Big).$$

(c) Translating the statement in (b) into C++ yields

```
if (x==0 || y/x >= 1)
cout << "True"
else
cout << "False".
```