Section 2.2

1 Determine the complexity function that measures the number of print statements in an algorithm that takes a positive integer n and prints one 1, two 2s, three 3s, $\ldots$, n ns.

Solution

$$f(n) = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} = O(n^2)$$

2. Suppose an algorithm takes a sequence of n (≥ 2) integers and determines if it contains an integer that is a repeat of the first integer in the list. Find the complexity function for the:

(a) best case analysis,

(b) worst case analysis,

(c) average case analysis.

Solution

(a) The complexity function for the best case is $f(n) = 1$. Making the second integer equal to the first will force the algorithm to terminate after only one comparison.

(b) The complexity function for the worst case is $f(n) = n$. Having no repeat of the first integer will force the algorithm to terminate after making all $n - 1$ comparisons.

(c) The complexity function for the average case is $f(n) = n$. There might be a repeat of the first integer in any of positions 2 through n, or there may be not repeat. Thus there are n cases, with respective numbers of comparisons $1, 2, 3, \ldots, n-1, n-1$. The average of these numbers is
$$\frac{1 + 2 + 3 + \cdots + (n-1) + (n-1)}{n} = \frac{n(n-1)/2}{n} = O(n).$$

3. Find the complexity function for counting the number of print statements in the following algorithm:

for $i := 1$ **to** n
begin
  for $j := 1$ **to** n
    print "hello"
  for $k := 1$ **to** n
    print "hello"
end

Solution For each value of i, both the j-loop and k-loop are executed. Thus for each i, $n + n = 2n$ print statements are executed. Therefore the total number of print statements executed is $n \cdot 2n = 2n^2 = O(n^2)$.

4. Find the complexity function for counting the number of print statements in the following algorithm:

   for $i := 1$ **to** n

   begin

     for $j := 1$ **to** i

       print "hello"

     for $k := i + 1$ **to** n

       print "hello"

   end

   Solution For each value of i, both the j-loop and k-loop are executed. Thus for each i, $i+j = n$ print statements are executed. Therefore the total number of print statements executed is $n \cdot n = O(n^2)$.