Homework 1 Cheat Sheet

Friday, April 6th, 2007

Truth Tables

Logic Operators								Bit Operations				
p	q	$\neg q$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$	x	y	$x \wedge y$	$x \vee y$	$x \oplus y$
T	T	F	T	T	F	T	T	1	1	1	1	0
T	F	T	F	T	T	F	F	1	0	0	1	1
F	T	–	F	T	T	T	F	0	1	0	1	1
F	F	–	F	F	F	T	T	0	0	0	0	0

Simple Equivalences

Identity:	Domination:	Idempotent:	Negation:	Implication:
$p \wedge T \Leftrightarrow p$ $p \vee F \Leftrightarrow p$	$p \vee T \Leftrightarrow T$ $p \wedge F \Leftrightarrow F$	$p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	$\neg(\neg p) \Leftrightarrow p$ $p \vee \neg p \Leftrightarrow T$ $p \wedge \neg p \Leftrightarrow F$	$(p \rightarrow q) \Leftrightarrow (\neg p \vee q)$

Equivalence Laws

Commutative:

$p \vee q \Leftrightarrow q \vee p$

$p \wedge q \Leftrightarrow q \wedge p$

Associative:

$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

De Morgan's:

$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Distributive:

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

$\mathbb{Z}$: Integers $(\ldots, -1, 0, 1, \ldots)$

$\mathbb{N}$: Natural Numbers $(1, 2, 3, \ldots)$

$\mathbb{Q}$: Rational Numbers ($\frac{p}{q}$ where $p, q \in \mathbb{Z}, q \neq 0$)

$\mathbb{R}$: Real Numbers (rattional & irrational numbers)

Quantifications

Statement:	$\forall x\ \forall y\ P(x,y) \Leftrightarrow \forall y\ \forall x\ P(x,y)$
True if:	$P(x,y)$ is true for every pair x, y.
False if:	There exists a pair x, y for which $P(x,y)$ is false.
Statement:	$\forall x\ \exists y\ P(x,y)$
True if:	For every x there exists a y for which $P(x,y)$ is true.
False if:	There exists an x such that $P(x,y)$ is false for every y.
Statement:	$\exists x\ \forall y\ P(x,y)$
True if:	There exists an x for which $P(x,y)$ is true for every y.
False if:	For every x, there exists a y for which $P(x,y)$ is false.
Statement:	$\exists x\ \exists y\ P(x,y) \Leftrightarrow \exists y\ \exists x\ P(x,y)$
True if:	There exists a pair x, y for which $P(x,y)$ is true.
False if:	$P(x,y)$ is false for every pair x, y.
Statement:	$\neg \exists x\ P(x) \Leftrightarrow \forall x\ \neg P(x)$
True if:	$P(x)$ is false for every x.
False if:	There exists an x for which $P(x)$ is true.
Statement:	$\neg \forall x\ P(x) \Leftrightarrow \exists x\ \neg P(x)$
True if:	There exists an x for which $P(x)$ is false.
False if:	$P(x)$ is true for every x.