Discussion Notes: Homework 1

Friday, April 6th, 2007

Announcements

Please visit the TA website at `wwwcsif.cs.ucdavis.edu/~engle/ecs20-s07/` for more resources. This includes how to access the newsgroups, turn in homework, and various notes from discussion.

Homework 1 is due on Monday by 4:00pm in the ECS20 homework box in Kemper Hall room 2131 (on the second floor). Please make sure to write legibly or type your homework. If I am unable to read your homework, you will get zero points.

Discussions will be used to cover homework-related questions and examples. Please come ready with questions!

Conjunctive and Disjunctive Normal Forms

Lets convert the statement $\neg(r \vee (q \wedge (\neg r \rightarrow \neg p)))$ to normal form using truth tables. (This is from the extra exercises for section 1.2.)

(1) Create a truth table for the statement.

p	q	r	$\neg r \rightarrow \neg p$	$q \wedge (\neg r \rightarrow \neg p)$	$r \vee (q \wedge (\neg r \rightarrow \neg p))$	$\neg(r \vee (q \wedge (\neg r \rightarrow \neg p)))$
T	T	T	T	T	T	F
T	T	F	F	F	F	T
T	F	T	T	F	T	F
T	F	F	F	F	F	T
F	T	T	T	T	T	F
F	T	F	T	T	T	F
F	F	T	T	F	T	F
F	F	F	T	F	F	T

Notice $\neg r \rightarrow \neg p$ is false whenever r is false and p is true.

(2) Examine the variable and final result columns.

(i) To convert into disjunctive normal form:

(i) For every row where there is a T in the final column:

- Include each variable that has a T in its column.
- Include the negation of each variable that has a F in its column.
- Form a conjunction out of these variables.

This gives us the conjunctions:

- $p \wedge q \wedge \neg r$
- $p \wedge \neg q \wedge \neg r$
- $\neg p \wedge \neg q \wedge \neg r$

(ii) Form the disjunction of the above conjunctions:
$(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$

(iii) To convert into conjunctive normal form:

(i) For every row where there is a F in the final column:

- Include each variable that has a T in its column.
- Include the negation of each variable that has a F in its column.

- Form a conjunction out of these variables.

This gives us the conjunctions:

- $p \wedge q \wedge r$
- $p \wedge \neg q \wedge r$
- $\neg p \wedge q \wedge r$
- $\neg p \wedge q \wedge \neg r$
- $\neg p \wedge \neg q \wedge r$

(ii) Form the disjunction of the above conjunctions:
$(p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

(iii) Take the negation of the above statement:
$\neg\big((p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)\big)$

(iv) Use De Morgan's laws to simplify into the conjunction of disjunctions:
$(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r)$

Let F the statement we want to convert into conjunctive normal form. We find the disjunctive normal form for $\neg F$, and then negate that statement ($\neg\neg F$) and use De Morgan's laws to find the conjunctive normal form for F.

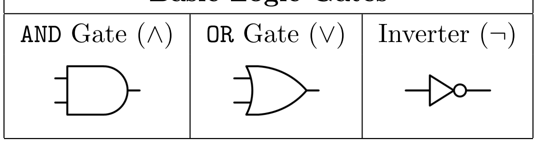
Once you start getting above three variables, this method becomes very cumbersome. You might instead be able to convert using the various equivalence laws in the book.

For example, consider $(p \wedge q \wedge r) \vee (s \wedge t)$:

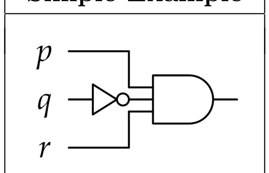
$$
\begin{aligned}
(p \wedge q \wedge r) \vee (s \wedge t) &\Leftrightarrow (p \vee (s \wedge t)) \wedge (q \vee (s \wedge t)) \wedge (r \vee (s \wedge t)) && \text{Distribute } s \wedge t \\
&\Leftrightarrow ((p \vee s) \wedge (p \vee r)) \wedge (q \vee (s \wedge t)) \wedge (r \vee (s \wedge t)) && \text{Distribute } p \\
&\Leftrightarrow ((p \vee s) \wedge (p \vee r)) \wedge ((q \vee s) \wedge (q \vee t)) \wedge (r \vee (s \wedge t)) && \text{Distribute } q \\
&\Leftrightarrow ((p \vee s) \wedge (p \vee r)) \wedge ((q \vee s) \wedge (q \vee t)) \wedge ((r \vee s) \wedge (r \vee t)) && \text{Distribute } r \\
&\Leftrightarrow (p \vee s) \wedge (p \vee r) \wedge (q \vee s) \wedge (q \vee t) \wedge (r \vee s) \wedge (r \vee t) && \text{Associative}
\end{aligned}
$$

Boolean Circuits

Basic Logic Gates

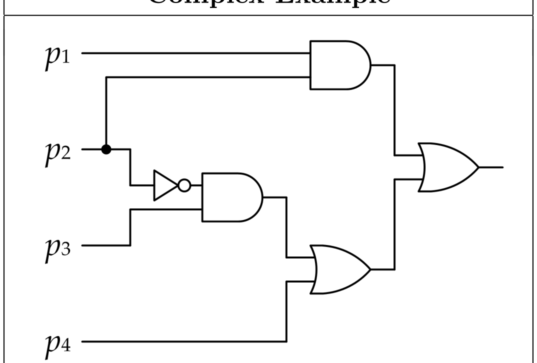


Simple Example



$\boldsymbol{p \wedge \neg q \wedge r}$

Complex Example



$\boldsymbol{(p_1 \wedge p_2) \vee \big((\neg p_2 \wedge p_3) \vee p_4\big)}$

Assume that you can use more than 2 inputs for your AND and OR gates.

Quantifiers

Assume $x, y \in \mathbb{R}$ unless otherwise specified.

Statement	**¬Statement**	**True Example**	**False Example**
$\forall x\ \forall y\, P(x,y)$	$\exists x\ \exists y\, \neg P(x,y)$	$\forall x\ \forall y\,(x+y>0)$ When $x, y \in \mathbb{N}$	$\forall x\ \forall y\,(x+y>0)$ When $x, y \in \mathbb{Z}$
$\forall x\ \exists y\, P(x,y)$	$\exists x\ \forall y\, \neg P(x,y)$	$\forall x\ \exists y\,(x \times y = 0)$ Let $y = 0$.	$\forall x\ \exists y\,(x \times y = 1)$ Breaks for $x = 0$.
$\exists x\ \forall y\, P(x,y)$	$\forall x\ \exists y\, \neg P(x,y)$	$\exists x\ \forall y\,(y \geq x)$ When $x, y \in \mathbb{N}$ and $x = 1$.	$\exists x\ \forall y\,(y \geq x)$ When $x, y \in \mathbb{Z}$.
$\exists x\ \exists y\, P(x,y)$	$\forall x\ \forall y\, \neg P(x,y)$	$\exists x\ \exists y\,(x+y<0)$ Let $x = 1$ and $y = -3$.	$\exists x\ \exists y\,(x+y<0)$ When $x, y \in \mathbb{N}$

Logic Problems

A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that:

(a) If the butler is telling the truth, then so is the cook.
(b) The cook and the gardener cannot both be telling the truth.
(c) The gardener and the handyman are not both lying.
(d) If the handyman is telling the truth then the cook is lying.

For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning. (1.1 # 41)

First, we should turn the "facts" into logical expressions.

Our variables are: b (for butler), c (for cook), g (for gardener), h (for handyman). Let the value of true indicate that the person is telling the truth. For example, $b = \mathrm{T}$ means the butler is telling the truth.

Then we can translate the statements as follows:

(1) $b \to c$ (2) $\neg(c \wedge g)$ (3) $\neg(\neg g \wedge \neg h)$ (4) $h \to \neg c$

We know all of these must hold. So we really want to know when $(b \to c) \wedge \neg(c \wedge g) \wedge \neg(\neg g \wedge \neg h) \wedge (h \to \neg c)$ is true. Behold, another truth table task! If you work out the truth table, you see that this statement is only true when:

b	c	g	h	$(b \to c) \wedge \neg(c \wedge g) \wedge \neg(\neg g \wedge \neg h) \wedge (h \to \neg c)$
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T

Therefore we can say the butler and cook are definitely lying, but we can not determine if the gardener or handyman are lying.

However, there are four variables in this problem. Working out a truth table is tedious! (Trust me, it was not fun.) Instead, we can do this with some straight reasoning. Notice that:

If the butler is telling the truth, then by (a) the cook is telling the truth.

If the cook is telling the truth, then by (b) the gardener is lying.

If the gardener is lying, then by (c) the handyman is telling the truth.

If the handyman is telling the truth, they by (d) the cook is lying.

This leads to a contradiction (the cook can not be both telling the truth and lying)! Therefore the butler and cook must be lying.

What about the gardener and the handyman? We don't have enough information to figure out if they are lying. Since the cook is lying, we can't use (b) to come to any conclusions about the gardener. Even if we are able to conclude that the gardener is telling the truth, we can't use (c) to come to any conclusions about the handyman.

Tautologies and Equivalences

It's true! You don't need truth tables to prove tautologies or equivalences. Instead, you can depend on the equivalences already established in the book.

For example, to prove that $(p \wedge q) \rightarrow p$ is a tautology:

$$\begin{aligned}
(p \wedge q) \rightarrow p &\Leftrightarrow \neg(p \wedge q) \vee p && \text{Implication} \\
&\Leftrightarrow (\neg p \vee \neg q) \vee p && \text{De Morgan} \\
&\Leftrightarrow (\neg p \vee p) \vee \neg q && \text{Assoc/Comm} \\
&\Leftrightarrow T \vee \neg q && (\neg p \vee p) \Leftrightarrow T \\
&\Leftrightarrow T && \text{Domination}
\end{aligned}$$

Translations

See the extra exercises provided for the book for examples on how to translate between propositions and English.

DISCLAIMER: This is meant to supplement the discussion section, not replace it!