DISCUSSION #5
FRIDAY MAY 4TH 2007

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ECS20: Discrete Mathematics

Discussion Outline

- Quick Announcements
- Induction Examples
  - Example 2 (page 268)
  - Example 7 (page 272)
  - Example 12 (page 276)

  - Note that I will be working these out on the board. If you want to see how they are done please see the book.

Announcements

- Homework 3:
  - Grades posted on my.ucdavis.edu.
- Homework 5:
  - Due Wednesday May 9th.
  - ~~Total of 7 problems from section 4.1.~~
- Midterm:
  - Was one problem that several students had problems with.
  - May reappear on final exam!

Midterm Notes

- Prove that $a \equiv b \bmod m$ implies that $ac \equiv bc \pmod{mc}$ for any integer c.

  - We know that $a \equiv b \bmod m$ means that $(a - b)$ is a multiple of m so $(a - b) = km$ for some k.

  - Therefore $ac - bc = (a - b)c = kmc$ is a multiple of mc.

  - The fact that $ac - bc$ is a multiple of mc is the definition what it means for ac to be congruent to $bc \pmod{mc}$.

Mathematical Induction

Chapter 4, Section 1

Mathematical Induction

- Step 0: Create a Conjecture
  - Figure out each step looks like based on n.
  - Figure out a pattern for the results based on n.
- Step 1: Basis Step
  - Show that $P(1)$ is true.
- Step 2: Inductive Step
  - Assume $P(k)$ is true.
  - Show that if $P(k)$ is true then $P(k + 1)$ is true.

Example 2

Conjecture a formula for the sum of the first n positive odd integers and prove your conjecture using mathematical induction.

Example 2

- Step 0: Create a Conjecture
  - Figure out each step looks like based on n.
  - Figure out a pattern for the results based on n.

Example 2

- Step 1: Basis Step
  - Show that $P(1)$ is true.

$P(1) = 1^2 = 1$ is true since the sum of the first one positive odd integer is 1.

(Notice the last term is $2n - 1 = 2 - 1 = 1$. Therefore we start and end with 1.)

Example 2

- Step 2: Inductive Step
  - Assume $P(k)$ is true.
  - Show that if $P(k)$ is true then $P(k+1)$ is true.

Homework 5

- For your homework, when proving by induction:
  - Always include the basis step.
  - Always state your assumption that $P(k)$ is true.
- These steps may seem unnecessary, but you do not have a proof by induction without them.

Example 7

- The harmonic numbers H_j for $j = 1, 2, 3, \ldots$, are defined by:

$$H_j = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{j} = \sum_{k=1}^{j} \frac{1}{k}$$

Use mathematical induction to show that:

$$H_{2^n} \geq 1 + \frac{n}{2}$$

whenever n is a nonnegative integer.

Example 7

- Step 1: Basis Step
  - Show that $P(1)$ is true.

Example 7

- Step 2: Inductive Step
  - Assume $P(k)$ is true.
  - Show that if $P(k)$ is true then $P(k+1)$ is true.

Example 7

- What does this mean?
  - The harmonic series is divergent!
    - Means that limit approaches infinity.

$$\sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

Example 12

- An odd number of people stand in a yard at **mutually distinct** distances. At the same time each person throws a pie at their nearest neighbor, hitting this person.

  Use mathematical induction to show that there is at least one survivor, that is, at least one person who is not hit by a pie.

Example 12

- Step 0: Create a Conjecture
  - We want to show that some conjecture $P(n)$ is true for all positive integers n.
  - We can't use n alone to represent the number of people in the fight, since n may be even.
  - Therefore we need to represent the number of people based on n such that:
    - The number of people is always odd.
    - There are at least 3 people.
      (a one person pie fight isn't very interesting)

Example 12

- Step 0: Create a Conjecture
  - Let $P(n)$ be the statement that for $2n + 1$ people in the pie fight, at least one person is not hit by a pie.

- Step 1: Basis Step
  - For $P(1)$ there are $2 * 1 + 1 = 3$ people.
    - Call these three Xander, Yasmin, and Zane.
  - Since each person is at mutually distinct distances, for each person there is only one closest neighbor.

Example 12

- Step 1: Basis Step
  - Suppose the shortest distance is between Xander and Yasmin.
    - Then Xander will throw a pie at Yasmin.
    - Then Yasmin will throw a pie at Xander.
  - Therefore, there is nobody to throw a pie at Zane.
    - Zane will throw a pie at the closest person.
  - Hence, for $P(1)$, one person will not be hit by a pie.

Example 2

- Step 2: Inductive Step
  - Assume $P(k)$ is true.
    - Assume for $2k+1$ people, one person is not hit by a pie.
  - Show that if $P(k)$ is true then $P(k+1)$ is true.
    - For $P(k+1)$, there are $2(k+1)+1 = 2k+3$ people.
    - Therefore there are also $2k+3$ pies to throw.
    - Let the shortest distance be between Xander & Yasmin.
      - Therefore, Xander throws a pie at Yasmin.
      - Therefore, Yasmin throws a pie at Xander.
    - There are now $2k+1$ pies to throw by $2k+1$ people.

Example 2

- Show that if $P(k)$ is true then $P(k+1)$ is true.
  - There are now $2k+1$ pies to throw by $2k+1$ people.
  - If Zane throws his pie at Xander or Yasmin then:
    - There are $2k$ pies left to throw at $2k+1$ people.
    - If there are less pies than people, someone must survive!
    - This is known as the pigeon hole principle.
  - If no one throws a pie at Xander or Yasmin then:
    - We now have $2k+1$ people.
    - By our assumption $P(k)$, for $2k+1$ people, at least one person is not hit by a pie.