■ Section 6.1 #16

Question:

What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit?

■ Section 6.1 #38

Question:

Two events E_1 and E_2 are called *independent* if

$$p(E_1 \cap E_2) = p(E_1)p(E_2)$$

For each of the following pairs of events, which are subsets of the set of all possible outcomes when a coin is tossed three times, determine whether or not they are independent.

(a) E_1: the first coin comes up tails;
E_2: the second coin comes up heads.

(b) E_1: the first coin comes up tails;
E_2: two, and not three, heads come up in a row.

(c) E_1: the second coin comes up tails;
E_2: two, and not three, heads come up in a row.

■ Section 6.2 #6

Question:

What is the probability of these events when we randomly select a permutation of $\{1, 2, 3\}$?

(a) 1 precedes 3

(b) 3 precedes 1

(c) 3 precedes 1 and 3 precedes 2

■ Section 6.2 #8

Question:
What is the probability of these events when we randomly select a permutation of $\{1, 2, \ldots, n\}$ where $n \geq 4$?

(a) 1 precedes 2
(b) 2 precedes 1
(c) 1 immediately precedes 2
(d) n precedes 1 and $n-1$ precedes 2
(e) n precedes 1 and n precedes 2

■ Section 6.2 #12

Question:

Suppose that E and F are events such that $p(E) = 0.8$ and $p(F) = 0.6$. Show that $p(E \cup F) \geq 0.8$ and $p(E \cap F) \geq 0.4$.

■ Section 6.2 #16

Question:

Show that if E and F are independent events, then $\overline{E}$ and $\overline{F}$ are also independent events.

■ Section 6.2 #18

Question:

Assume that the year has 366 days and all birthdays are equally likely.

(a) What is the probability that two people chosen at random were born on the same day of the week?

(b) What is the probability that in a group of n people chosen at random, there are at least two born on the same day of the week?

(c) How many people chosen at random are needed to make the probability greater than $1/2$ that there are at least two people born on the same day of the week?

■ Section 6.2 #20

Question:

Assume that the year has 366 days and all birthdays are equally likely.

Find the smallest number of people you need to choose at random so that the probability that at least one of them were both born on April 1 exceeds 1/2.

■ Section 6.2 #26

Question:

Let E be the event that a randomly generated bit string of length three contains an odd number of 1s, and let F be the event that the string starts with 1. Are E and F independent?

■ Section 6.2 #34

Question:

Find each of the following probabilities when n independent Bernoulli trails are carried out with probability of success p.

(a) the probability of no success

(b) the probability of at least one success

(c) the probability of at most one success

(d) the probability of at least two successes

■ Section 6.4 #4

Question:

A coin is biased so that the probability a head comes up when it is flipped is 0.6. What is the expected number of heads that come up when it is flipped 10 times?

■ Section 6.4 #6

Question:

What is the expected value when a \$1 lottery ticket is bought in which the purchaser wins exactly \$10 million if the ticket contains the six winning numbers chosen from the set $\{1, 2, \ldots, 50\}$ and the purchaser wins nothing otherwise?

■ Section 6.4 #8

Question:
What is the expected sum of the numbers that appear when three fair dice are rolled?

■ Section 6.4 #10

Question:

Suppose that we flip a coin until either it comes up tails twice or we have flipped it six times. What is the expected number of times we flip the coin?

■ Section 6.4 #12

Question:

Suppose that we roll a die until a 6 comes up.

(a) What is the probability that we roll the die n times?

(b) What is the expected number of times we roll the die?

■ Section 6.4 #16

Question:
Let X and Y be the random variables that count the number of heads and the number of tails that come up when two coins are flipped. Show that X and Y are not independent.

■ Section 6.4 #20

Question:
Let A be an event. Then I_A, the *indicator random variable* of A, equals 1 if A occurs and equals 0 otherwise.

Show that the expectation of the indicator random variable of A equals the probability of A, that is $E(I_A) = p(A)$.

■ Section 6.4 #24

Question:

What is the variance of the number of times a 6 appears when a fair dice is rolled 10 times?

■ Section 6.4 #30

Question:

Use Chebyshev's Inequality to find an upper bound on the probability that the number of tails that come up when a biased coin with probability of heads equal to 0.6 is tossed n times deviates from the mean by more than $\sqrt{n}$.